

15.5, number 19: Compute

$$\int_0^{\pi/4} \int_0^{\ln \sec v} \int_{-\infty}^{2t} e^x dx dt dv.$$

Answer:

$$\begin{aligned} & \int_0^{\pi/4} \int_0^{\ln \sec v} \int_{-\infty}^{2t} e^x dx dt dv \\ &= \int_0^{\pi/4} \int_0^{\ln \sec v} \lim_{b \rightarrow -\infty} e^x \Big|_b^{2t} dt dv \\ &= \int_0^{\pi/4} \int_0^{\ln \sec v} \lim_{b \rightarrow -\infty} (e^{2t} - e^b) dt dv \\ &= \int_0^{\pi/4} \int_0^{\ln \sec v} e^{2t} dt dv \\ &= \int_0^{\pi/4} \frac{1}{2} e^{2t} \Big|_0^{\ln \sec v} dv \\ &= \int_0^{\pi/4} \frac{1}{2} (e^{2 \ln \sec v} - 1) dv \\ &= \int_0^{\pi/4} \frac{1}{2} (\sec^2 v - 1) dv \\ &= \frac{1}{2} (\tan v - v) \Big|_0^{\pi/4} \\ &= \boxed{\frac{1}{2} \left(1 - \frac{\pi}{4}\right)} \end{aligned}$$