15.4, number 9: **Change the integral into polar coordinates, then integrate:**

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \, dy \, dx.$$

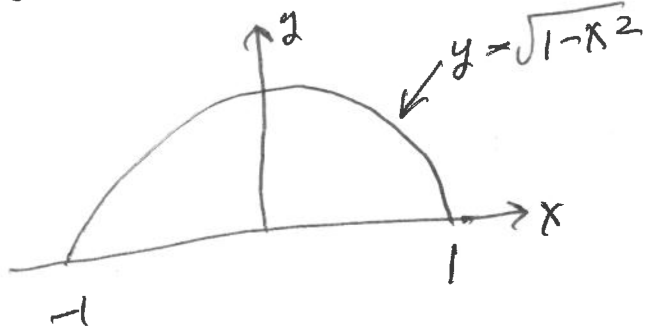
Answer: The given integral is taken over the top half of the circle with radius 1 and center $(0, 0)$. (See, the next page, if necessary.) Keep in mind the $dy \, dx$ becomes $r \, dr \, d\theta$.

$$\begin{aligned}
& \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \, dy \, dx \\
= & \int_{0}^{\pi} \int_{0}^{1} r \, dr \, d\theta \\
= & \int_0^{\pi} \left. \frac{r^2}{2} \right|_0^1 d\theta \\
= & \int_0^{\pi} \frac{1}{2} \, d\theta \\
= & \left. \frac{1}{2}\theta \right|_0^{\pi} \\
= & \boxed{\frac{\pi}{2}}
\end{aligned}$$

Picture 15.4 Number 9

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\,dx$$

integrates over the region



square $y=\sqrt{1-x^2}$ to get $y^2=1-x^2$ or $x^2+y^2=1$

In polar coordinates the region is

For each fixed angle between 0 and π

r goes from 0 to 1