15.4, number 5: **Describe the region (drawn on the next page) in polar coordinates.**

Answer: Draw the line segment from the origin to the point $(2\sqrt{3}, 2)$. This line segment divides the region into two pieces. We deal with these pieces separately; we refer to these pieces as the lower piece and the upper piece.

All line segments, emanating from the origin, which touch the lower piece, start touching the lower piece when $r = 1$ and stop touching the lower piece when $x = 2\sqrt{3}$.

All line segments, emanating from the origin, which touch the upper piece, start touching the upper piece when $r = 1$ and stop touching the upper piece when $y = 2$.

Of course, $x = r\cos\theta$; so $x = 2\sqrt{3}$ when $r\cos\theta = 2\sqrt{3}$. In other words, in the lower piece $1 \le r \le \frac{2\sqrt{3}}{\cos\theta}$.

Similarly, $y = r\sin\theta$; so $y = 2$ when $r\sin\theta = 2$. In other words, in the upper piece $1 \le r \le \frac{2}{\sin\theta}$.

Now we know what the r's are doing. We better figure out what the θ's are doing. The cut off is the angle in the right triangle with opposite equal 2, adjacent equal to $2\sqrt{3}$. The Pythagorean Theorem says that the hypotenuse is 4. We are thinking about the angle whose cosine is $\frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$. So the cut off angle is $\frac{\pi}{6}$.

The lower region is described by saying "for all fixed θ with $0 \le \theta \le \frac{\pi}{6}$, r goes from 1 to $\frac{2\sqrt{3}}{\cos\theta}$".

The upper region is described by saying "for all fixed θ with $\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$, r goes from 1 to $\frac{2}{\sin\theta}$".

Picture 15.4 Number 5

Original Picture