15.4, number 25: **Convert the integral into an integral involving $dx$ and $dy$. Do Not compute any integral.**

$$\int_0^{\pi/4} \int_0^{2\sec\theta} r^5 \sin^2\theta \, dr \, d\theta.$$

**Answer:** The integral is taken over the region described by: "For each fixed $\theta$ with $0 \le \theta \le \frac{\pi}{4}$, $r$ goes from $r = 0$ to $r = 2\sec\theta$." Of course, $\sec\theta = \frac{1}{\cos\theta}$. (Keep in mind that $0 \le \theta \le \frac{\pi}{4}$; so $\cos\theta$ is positive.) Multiply all sides of the bound $0 \le r \le 2\sec\theta$ by the positive number $\cos\theta$ to obtain $0 \le r\cos\theta \le 2$ or

$$0 \le x \le 2.$$

The tangent function is increasing for $0 \le \theta \le \frac{\pi}{4}$. Apply $\tan$ to the inequality $0 \le \theta \le \frac{\pi}{4}$ to learn

$$\underbrace{\tan 0}_{0} \le \underbrace{\tan\theta}_{\frac{y}{x}} \le \underbrace{\tan\frac{\pi}{4}}_{1}.$$

Multiply by $x$, which is positive. The outer bound translates to

$$0 \le y \le x.$$

The region of the old integral is described by

$$\int_0^2 \int_0^x * \, dy \, dx.$$

Now we translate $*$. Well, $r\,dr\,d\theta = dy\,dx$, $r^2\sin^2\theta = y^2$, and $r^2 = x^2 + y^2$. The old integral is equal to

$$\boxed{\int_0^2 \int_0^x y^2(x^2+y^2)\, dy\, dx.}$$