15.4, number 23: **Convert the integral into an integral involving dx and dy. Do Not compute any integral.**

$$\int_0^{\pi/2} \int_0^1 r^3 \sin\theta \cos\theta \, dr \, d\theta$$

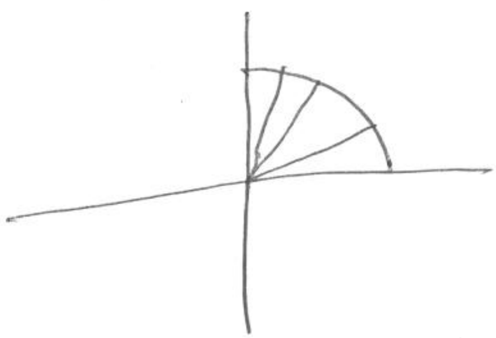
Answer: We see on the next page that the present integral is equal to

$$\boxed{\int_0^1 \int_0^{\sqrt{1-x^2}} xy \, dy \, dx.}$$

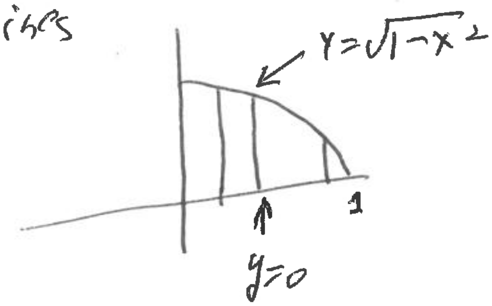
Picture 15.4 Number 23

The Picture for $\int_0^{\frac{\pi}{2}} \int_0^1 r^3 \sin\theta \cos\theta \, dr \, d\theta$

For each fixed θ with $0 \le \theta \le \frac{\pi}{2}$, r goes from $r=0$ to $r=1$:



In rectangalar coordinates we can fill the region with vertical lines



We view $r^3 \sin\theta \cos\theta \, dr \, d\theta$ as $\underbrace{(r\sin\theta)}_{y}\underbrace{(r\cos\theta)}_{x}\underbrace{(r\,dr\,d\theta)}_{dy\,dx}$

The original integral is

$$\int_0^1 \int_0^{\sqrt{1-x^2}} xy \, dy \, dx$$