15.4, number 21: **Change the integral into polar coordinates, then integrate:**

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y)\,dy\,dx$$

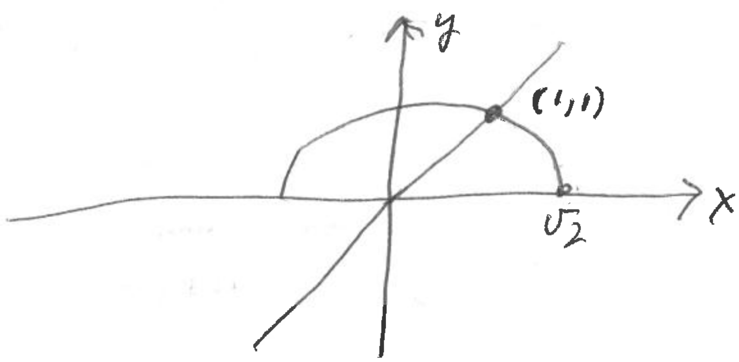
Answer: Look at the picture on the next page. The region for this integral is defined by: For each fixed θ with $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, r goes from $r=0$ to $r=\sqrt{2}$. In polar coordinates x becomes $r\cos\theta$ and y becomes $r\sin\theta$, and $dy\,dx$ becomes $r\,dr\,d\theta$. The original integral is equal to

$$\begin{aligned}
& \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} (r\cos\theta + 2r\sin\theta) r\,dr\,d\theta \\
= & \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} (r^2\cos\theta + 2r^2\sin\theta)\,dr\,d\theta \\
= & \int_{\pi/4}^{\pi/2} \left(\frac{r^3}{3}\cos\theta + \frac{2r^3}{3}\sin\theta\right)\Big|_0^{\sqrt{2}}\,d\theta \\
= & \int_{\pi/4}^{\pi/2} \left(\frac{2^{3/2}}{3}\cos\theta + \frac{2(2^{3/2})}{3}\sin\theta\right)d\theta \\
= & \left(\frac{2^{3/2}}{3}\sin\theta - \frac{2(2^{3/2})}{3}\cos\theta\right)\Big|_{\pi/4}^{\pi/2} \\
= & \frac{2^{3/2}}{3} - 0 - \frac{2^{3/2}}{3}\frac{\sqrt{2}}{2} + \frac{2(2^{3/2})}{3}\frac{\sqrt{2}}{2} \\
= & \boxed{\frac{2^{3/2}+2}{3}}
\end{aligned}$$

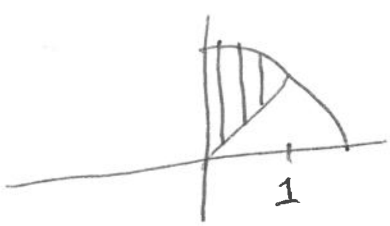
Picture 15.4 Number 21

The picture for $\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y)\,dy\,dx$

We draw $y=x$ and $y=\sqrt{2-x^2}$, Of course $y=\sqrt{2-x^2}$ is the part of the circle $x^2+y^2=2$ where y is positive



For each fixed x with $0 \le x \le 1$, y goes from $y=x$ to y on the circle



Of course this region is also described by

For each fixed θ with $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, r goes from $r=0$ to $r=\sqrt{2}$.