15.3, number 3: **Find the area of the region bounded by $x = -y^2$ and $y = x + 2$.**

Answer:

The picture on the next page shows that best way to fill the region is with horizontal lines. For each fixed y with $-2 \le y \le 1$, x goes from $x = y - 2$ to $x = -y^2$. The area is equal to

$$\int_{-2}^{1} \int_{y-2}^{-y^2} dx\, dy$$

$$= \int_{-2}^{1} x \Big|_{y-2}^{-y^2} dy$$

$$= \int_{-2}^{1} (-y^2 - (y - 2))\, dy$$

$$= \int_{-2}^{1} (-y^2 - y + 2)\, dy$$

$$= \left(\tfrac{-y^3}{3} - \tfrac{y^2}{2} + 2y\right)\Big|_{-2}^{1}$$

$$= -\tfrac{1}{3} - \tfrac{1}{2} + 2 - \left(\tfrac{8}{3} - \tfrac{4}{2} - 4\right)$$

$$= -\tfrac{9}{3} - \tfrac{1}{2} + 2 + 2 + 4$$

$$= 5 - \tfrac{1}{2}$$

$$= \boxed{\tfrac{9}{2}}$$

Picture   15.3 Number 3

$x = y - 2$

$x = -y^2$

$(-1, 1)$

$(-4, -2)$