15.3, number 10: **Find the area of the region bounded by $y = 1 - x$, $y = 2$, and $y = e^x$.**

Answer: The picture on the next page shows that we should use horizontal lines to fill the region and that for each fixed y, with $1 \leq y \leq 2$, x goes from $x = 1 - y$ to $x = \ln y$. The area is

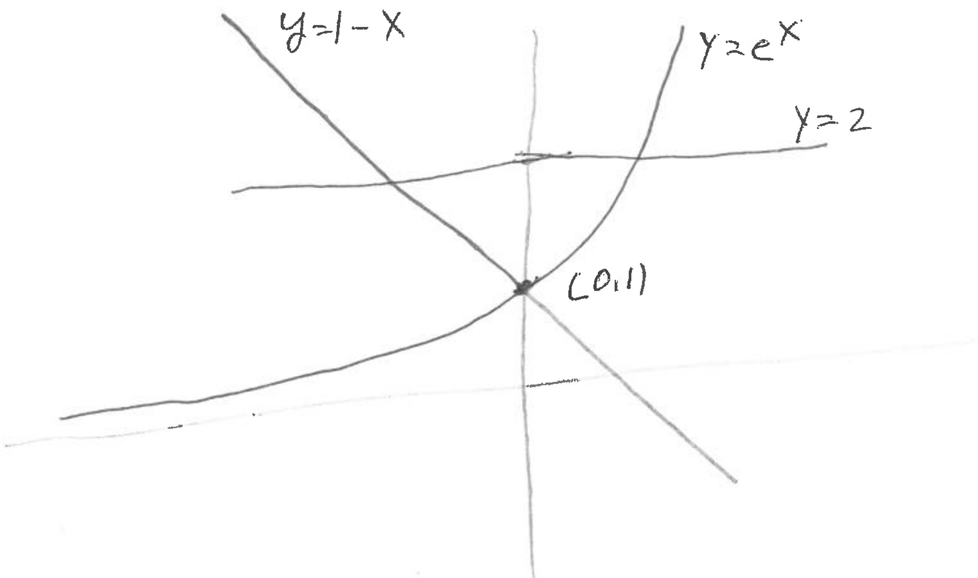
$$\begin{aligned} & \int_1^2 \int_{1-y}^{\ln y} dx\, dy \\ = & \int_1^2 x\Big|_{1-y}^{\ln y} dy \\ = & \int_1^2 \ln y - (1 - y)\, dy \end{aligned}$$

Recall that one uses integration by parts $\int u\, dv = uv - \int v\, du$ to compute $\int \ln y\, dy$. Take $u = \ln y$ and $dv = dy$. Compute $du = \frac{1}{y}\, dy$ and $v = y$. So, $\int \ln y\, dy = \int udv = uv - \int vdu = y \ln y - \int dy = y \ln y - y$.

$$\begin{aligned} & = (y \ln y - y - y + \tfrac{y^2}{2})\Big|_1^2 \\ & = (2 \ln 2 - 2 - 2 + \tfrac{2^2}{2}) - (1 \ln 1 - 1 - 1 + \tfrac{1^2}{2}) \\ & = (2 \ln 2 - 4 + 2) - (-2 + \tfrac{1}{2}) = \boxed{2 \ln 2 - \tfrac{1}{2}} \end{aligned}$$

Picture 15.3 Number 10

The region bounded by $y=1-x$, $y=2$, $y=e^x$ is



Fill the region with horizontal lines