15.2, number 60: **Sketch the region of integration for**

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dy\, dx.$$

Set up the integral over the same region, with the order of integration reversed. Evaluate the new integral.

Answer: This problem is an important problem. None of us know the anti-derivative of e^{y^3}. So none of us can do the problem as written. However, we are able to do the problem after reversing the order of integration. The given integral is taken over the region described as follows: for each fixed x with $0 \le x \le 3$, y goes from $y = \sqrt{\frac{x}{3}}$ to $y = 3$. There is a picture of the region on the page. The given integral fills the region with vertical lines. Of course, we can fill the region with horizontal lines. You must look at the picture to see this. For each fixed y, with $0 \le y \le 1$, x goes from $x = 0$ to $x = 3y^2$.

At any rate,

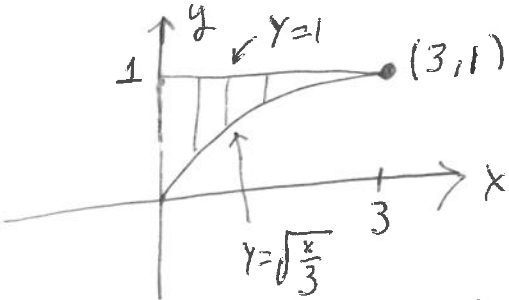
$$\begin{aligned}
&\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dy\, dx \\
&= \int_0^1 \int_0^{3y^2} e^{y^3}\, dx\, dy \\
&= \int_0^1 e^{y^3} x \Big|_0^{3y^2} dy \\
&= \int_0^1 3y^2 e^{y^3}\, dy \\
&= e^{y^3}\Big|_0^1 \\
&= \boxed{e-1}
\end{aligned}$$

Picture 15.2 Number 60

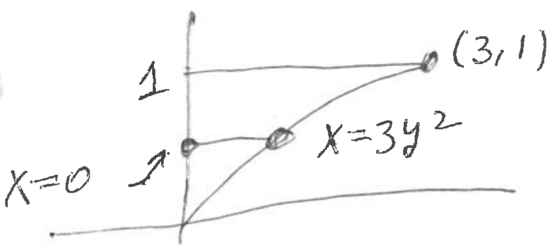
$\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3}\, dy\, dx$ is taken over the region described

by: For each fixed x with $0 \le x \le 3$, y goes from $y = \sqrt{\frac{x}{3}}$ to $y = 1$

Of course, $y = \sqrt{\frac{x}{3}}$ is the upper part of $3y^2 = x$



If we fill the region with horizontal lines we have



For each fixed y with $y = 0$ to $y = 1$, x goes from $x = 0$ to $x = 3y^2$

The old integral equals

$$\int_0^1 \int_0^{3y^2} e^{y^3}\, dx\, dy$$