

15.2, number 49: Sketch the region of integration for

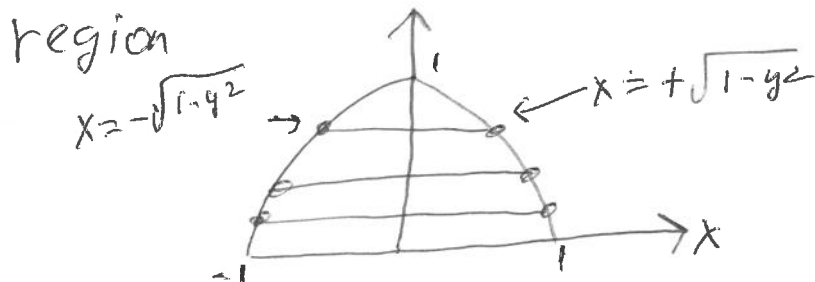
$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \, dx \, dy.$$

Set up the integral over the same region, with the order of integration reversed.

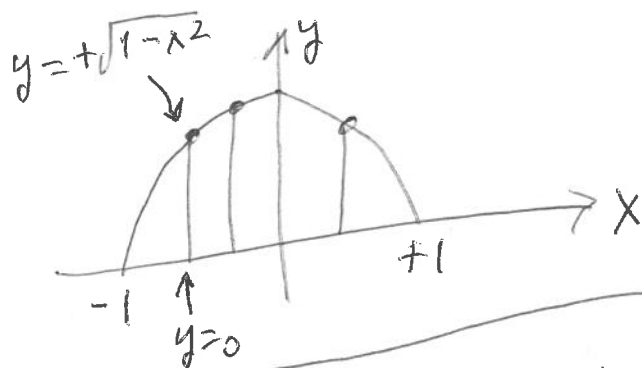
Answer: The given integral is taken over the region defined as follows: for each fixed y with $0 \leq y \leq 1$, x goes from $x = -\sqrt{1-y^2}$ to $x = +\sqrt{1-y^2}$. We draw this on the next page. Keep in mind that $x = -\sqrt{1-y^2}$ is the left half of the circle $x^2 + y^2 = 1$ and $x = +\sqrt{1-y^2}$ is the right half of the same circle. Please look at the next page.

Picture 15.2 Number 49

The integral $\int_0^1 \int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} 3y \, dx \, dy$ is taken over the



One can fill the same region with vertical lines:



The old integral is equal to

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} 3y \, dy \, dx$$