15.2, number 35: **Integrate** $f(u,v) = v - \sqrt{u}$ **over the triangular region in the first quadrant of the** uv**-plane cut out by** $u + v = 1$**.**

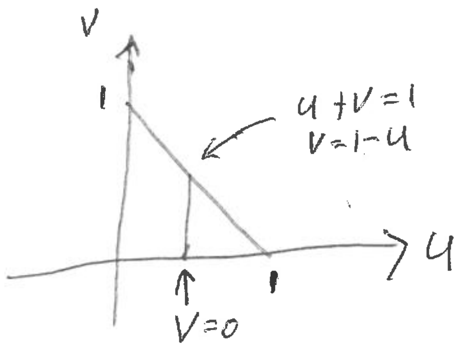
Answer: The picture is on the next page. We filled up the region using vertical lines. For each fixed u with $0 \le u \le 1$, v goes from 0 to $1 - u$.

$$
\begin{aligned}
&\int_0^1 \int_0^{1-u} (v - \sqrt{u})\, dv\, du \\
= &\textstyle\int_0^1 \left(\frac{v^2}{2} - \sqrt{u}v\right)\Big|_0^{1-u} du \\
= &\textstyle\int_0^1 \left(\frac{(1-u)^2}{2} - \sqrt{u}(1-u)\right) du \\
= &\textstyle\int_0^1 \left(\frac{(1-u)^2}{2} - \sqrt{u} + u^{3/2}\right) du
\end{aligned}
$$

I did a substitution when I integrated $(1-u)^2$. I let $w = 1 - u$. In this case $dw = -du$. etc. Do notice that the derivative of $-\frac{(1-u)^3}{6}$ is exactly $\frac{(1-u)^2}{2}$.

$$
\begin{aligned}
&= \left(-\tfrac{(1-u)^3}{6} - \tfrac{2}{3}u^{3/2} + \tfrac{2}{5}u^{5/2}\right)\Big|_0^1 \\
&= -\tfrac{2}{3} + \tfrac{2}{5} + \tfrac{1}{6} = \boxed{\tfrac{-1}{10}}.
\end{aligned}
$$

Picture 15.2 Number 35



For each fixed u from u = 0 to u = 1
v goes from 0 to 1 − u