15.2, number 33: **Integrate $f(x,y) = x/y$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$, and $x = 2$.**

Answer: There is a picture on the next page. We fill up the region with vertical lines. For each fixed x between 1 and 2, y goes from x to $2x$.
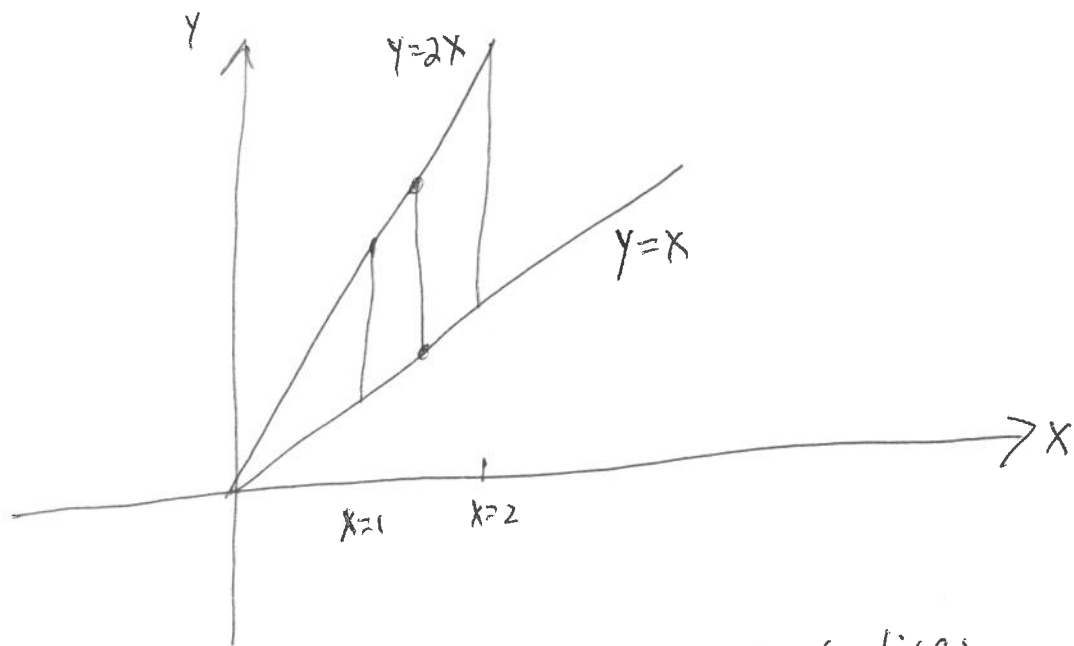
The integral is

$$\int_1^2 \int_x^{2x} \frac{x}{y}\, dy\, dx = \int_1^2 x(\ln y)\Big|_x^{2x}\, dx$$

$$= \int_1^2 x(\ln(2x) - \ln(x))\, dx$$

You could use integration by parts to calculate $\int x \ln x\, dx$; but it would be much more clever to use $\ln(ab) = \ln a + \ln b$. You can check this formula by exponentiating both sides.

$$= \int_1^2 x(\ln(2))\, dx$$

$$= \frac{x^2}{2} \ln 2 \Big|_1^2$$

$$= (2 - \tfrac{1}{2}) \ln 2$$

$$= \boxed{\frac{3 \ln 2}{2}}$$

Picture 15, 2   Number 33

We fill in the region using vertical lines
For each fixed X between X=1 and X=2
y goes from X to 2X