

15.2, number 23: **Compute** $\int_0^{\sqrt{\pi}} \int_0^{x^2} x \sin y \, dy \, dx$.

Answer:

$$\begin{aligned} \int_0^{\sqrt{\pi}} \int_0^{x^2} x \sin y \, dy \, dx &= \int_0^{\sqrt{\pi}} -x \cos y \Big|_0^{x^2} \, dx \\ &= \int_0^{\sqrt{\pi}} (-x \cos(x^2) + x) \, dx \\ &= \left(-\frac{1}{2} \sin(x^2) + \frac{1}{2}x^2 \right) \Big|_0^{\sqrt{\pi}} \\ &= -\frac{1}{2} \sin(\pi) + \frac{\pi}{2} \\ &= \boxed{\frac{\pi}{2}} \end{aligned}$$