

15.1, number 30: Find the volume of the region bounded above by the elliptical paraboloid  $z = 16 - x^2 - y^2$  and bounded below by the square  $0 \leq x \leq 2$  and  $0 \leq y \leq 2$ .

**Answer:** I filled up the square base using vertical lines. The volume is the integral over the base of the top, which is equal to

$$\begin{aligned} \int_0^2 \int_0^2 (16 - x^2 - y^2) dy dx &= \int_0^2 (16y - x^2y - \frac{y^3}{3}) \Big|_0^2 dx \\ &= \int_0^2 32 - 2x^2 - \frac{8}{3} dx \\ &= (32x - \frac{2}{3}x^3 - \frac{8}{3}x) \Big|_0^2 \\ &= 64 - \frac{16}{3} - \frac{16}{3} \\ &= 16(4 - \frac{2}{3}) = \boxed{\frac{160}{3}} \end{aligned}$$