14.7, S19-e3-4: **Find the absolute maximum and absolute minimum of**

$$f(x,y) = 2 + 2x + 4y - x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$**,** $y = 0$**, and** $y = 9 - x$**.**

Answer:
See the picture page for a picture of the domain.

We find all points in the interior where both partials vanish. Observe that $f_x = 2 - 2x$ and $f_y = 4 - 2y$. Both partials vanish at $\underline{(1,2)}$. This point is in the interior.

We find all points on $x = 0$ with $0 \le y \le 9$ where the derivative of $f(0,y) = 2 + 4y - y^2$ is zero. The derivative is $4 - 2y$. This is zero when $y = 2$. We must study $\underline{(0,2)}$.

We find all points on $y = 0$ with $0 \le x \le 9$ where the derivative of $f(x,0) = 2 + 2x - x^2$ is zero. The derivative is $2 - 2x$. The derivative is zero when $x = 1$. We must study $\underline{(1,0)}$.

We find all points on $y = 9 - x$ with $0 \le x \le 9$ where the derivative of $f(x, 9-x) = 2 + 2x + 4(9-x) - x^2 - (9-x)^2$ is zero. The derivative is $2 - 4 - 2x + 2(9-x) = 16 - 4x$. The derivative is zero when $x = 4$. We must study $\underline{(4,5)}$.

We must also study the end points of the boundary; namely, $\underline{(0,0)}$, $\underline{(0,9)}$, and $\underline{(9,0)}$.

We plug all 7 candidates into f:

$$\begin{aligned}
f(1,2) &= 2 + 2 + 8 - 1 - 4 = 7\\
f(0,2) &= 2 + 8 - 4 = 6\\
f(1,0) &= 2 + 2 - 1 = 3\\
f(4,5) &= 2 + 8 + 20 - 16 - 25 = -11\\
f(0,0) &= 2\\
f(0,9) &= 2 + 36 - 81 = -43\\
f(9,0) &= 2 + 18 - 81 = -61
\end{aligned}$$

The maximum point of f on the given domain is $(1, 2, 7)$.
The minimum point of f on the given domain is $(9, 0, -61)$.