14.7, F18-e3-2: **Find the absolute maximum and the absolute minimum values of**

$$f(x,y) = 3xy - 6x - 3y + 7$$

on the triangular region with vertices $(0,0)$, $(3,0)$**, and** $(0,5)$**.**

Answer:
We compute $\frac{\partial f}{\partial x} = 3y - 6$ and $\frac{\partial f}{\partial y} = 3x - 3$. Both partial derivatives are zero at the point $(x,y) = (1,2)$. This interior critical point might be an absolute extreme point.

We look at the restriction of f to the line $y = 0$ with $0 \leq x \leq 3$.

$$f|_{y=0}(x) = -6x + 7$$

and the derivative is -6 which is never zero. So the absolute extreme points of the restriction of f to the line $y = 0$ with $0 \leq x \leq 3$ occur at the end points.

We look at the restriction of f to the line $x = 0$ with $0 \leq y \leq 5$.

$$f|_{x=0}(y) = -3y + 7$$

and the derivative is -3 which is never zero. So the absolute extreme points of the restriction of f to the line $x = 0$ with $0 \leq y \leq 5$ occur at the end points.

The line which connects $(3,0)$ to $(0,5)$ is $y = -\frac{5}{3}x + 5$. We look at the restriction of f to the line $y = -\frac{5}{3}x + 5$ with $0 \leq x \leq 3$.

$$\begin{aligned} f|_{\text{top boundary}}(x) &= 3x(-\frac{5}{3}x + 5) - 6x - 3(-\frac{5}{3}x + 5) + 7 \\ &= -5x^2 + 15x - 6x + 5x - 15 + 7. \end{aligned}$$

The derivative is $-10x+15-6+5$. The derivative is zero when $-10x+14 = 0$; that is when $x = 7/5$ and $y = 8/3$.

The extreme points of f on our domain occur at $(0,0)$, $(3,0)$, $(0,5)$, or $(7/5, 8/3)$. We plug these points into f:

$$\begin{aligned} f(0,0) &= 7, && \text{absolute maximum} \\ f(3,0) &= -18 + 7 = -11 && \text{absolute minimum} \\ f(0,5) &= -15 + 7 = -8 \\ f(7/5, 8/3) &= 56/5 - 42/5 - 8 + 7 = 9/5 \\ f(1,2) &= 6 - 6 - 6 + 7 = 1 \end{aligned}$$

The absolute minimum of f on our domain occurs at $(3,0,-11)$.
The absolute maximum of f on our domain occurs at $(0,0,7)$.