14.7, F18-e3-1: **Find all local minima, local maxima and saddle points for the function** $f(x,y) = x^2 + 4y^2 - 6x + 8y - 15$**.**

Answer:
We compute $\frac{\partial f}{\partial x} = 2x - 6$ and $\frac{\partial f}{\partial y} = 8y + 8$. Both partial derivatives are zero at the point $(x,y) = (3,-1)$. We apply the second derivative test at that point. We compute $\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y \partial x} = 0$, and $\frac{\partial^2 f}{\partial y^2} = 8$.

Observe that

$$\left(\frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial y \partial x} \right)^2 \right) \Bigg|_{(x,y)=(3,-1)} = 16,$$

which is positive. Thus, $(3,-1,f(3,1))$ is not a saddle point; it is either a local maximum or a local minimum. Also

$$\frac{\partial^2 f}{\partial x^2} \Bigg|_{(x,y)=(3,-1)} = 2,$$

which is positive. We conclude that

$$\boxed{(3,-1,f(3,1)) \text{ is a local minimum.}}$$