14.7, F17-e3-11:40-section-4: **Find the absolute maximum and minimum values of**

$$f(x, y) = 2 + 2x + 4y - x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$**,** $y = 0$**, and** $y = 9 - x$**.**

Answer:
We drew the region on the picture page.

- We first look for interior points where f_x and f_y both vanish. We compute $f_x = 2 - 2x$ and $f_y = 4 - 2y$. We see that the only point where $f_x = 0$ and $f_y = 0$ is $\underline{(1, 2)}$.
- We find points on $x = 0$ where the derivative of (f restricted to $x = 0$) vanishes. We are interested in $f(y) = 2 + 4y - y^2$ with $0 \leq y \leq 9$. We compute $f'(y) = 4 - 2y$. We consider $\underline{(0, 2)}$.
- We find points on $y = 9 - x$ where the derivative of (f restricted to $y = 9 - x$) vanishes. We are interested in

  $$f(x) = 2 + 2x + 4(9 - x) - x^2 - (9 - x)^2$$

  with $0 \leq x \leq 9$. We compute $f'(x) = 2 - 4 - 2x + 2(9 - x) = 16 - 4x$. We consider $\underline{(4, 5)}$.
- We find points on $y = 0$ where the derivative of (f restricted to $y = 0$) vanishes. We are interested in $f(x) = 2 + 2x - x^2$ for $0 \leq x \leq 9$. We compute $f'(x) = 2 - 2x$. We consider $\underline{(1, 0)}$.
- We also consider the corner points $\underline{(0, 0)}$, $\underline{(0, 9)}$, $\underline{(9, 0)}$.

The maximum and the minimum of f occur at one of the underlined points. We compute f at each of these points:

$$\begin{aligned} &f(1, 2) = 7 \\ &f(0, 2) = 6 \\ &f(4, 5) - 11 \\ &f(1, 0) = 3 \\ &f(0, 0) = 2 \\ &f(0, 9) = -43 \\ &f(9, 0) = -61 \end{aligned}$$

Thus,

The maximum of f on the given region is 7 and $f(1, 2) = 7$. The minimum of f on the given region is -61 and $f(9, 0) = -61$.