14.7, number 7: **Find all local maxima, local minima, and saddle points of $f(x, y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$.**

Answer: We compute

$$\begin{aligned} f_x =& 4x + 3y - 5 \\ f_y =& 3x + 8y + 2 \\ f_{xx} =& 4 \\ f_{xy} =& 3 \\ f_{yy} =& 8 \end{aligned}$$

We see that f_x and f_y are zero when both equations:

$$\begin{cases} 4x + 3y - 5 = 0 \\ 3x + 8y + 2 = 0 \end{cases} \tag{2}$$

are satisfied. The solution set is not changed if we replace Equation 2 by Equation 2 minus $\frac{3}{4}$ times Equation 1:

$$\begin{cases} 4x + 3y - 5 = 0 \\ \qquad \frac{23}{4}y + \frac{23}{4} = -0 \end{cases}$$

The second equation tells us that $y = -1$ so the first equation tells us that $x = 2$. (Be sure to check that $(2, 1)$ really is the solution of (2).)

We apply the Second Derivative Test at $(2, 1)$. We calculate

$$(f_{xx}f_{yy} - (f_{xy})^2)|_{(2,1)} = 4(8) - 9,$$

which is positive. We see that $f_{xx}|_{(2,1)} = 4$, which is also positive. We conclude that

$$\boxed{(2, 1, f(2, 1)) \text{ is a local minimum of the graph of } z = f(x, y).}$$