14.7, number 33: **Find the absolute maxima and absolute minima of $f(x,y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0$, $y = 0$, $y + 2x = 2$ in the first quadrant.**

Answer: There is a picture of the region on the next page. Observe that $\underline{(0,0)}$, $\underline{(0,2)}$, and $\underline{(1,0)}$ are end points of the boundary. These are points of interest. After we have found all of the points of interest, we will evaluate f at each of these points.

Any point in the region where both partial derivatives are zero is a point of interest. We see that $f_x = 2x$ and $f_y = 2y$. So $(0,0)$ is the only point in the region where both partial derivatives are zero. This is a point of interest. (Of course, we already know about this point.)

Now we look at f restricted to each part of the boundary.

The restriction of f to $x = 0$ is

$$f|_{x=0}(y) = y^2.$$

We compute

$$\frac{d}{dy}(f|_{x=0}) = 2y.$$

This derivative is zero when $y = 0$. The corresponding point in our region is $(0,0)$. This is a point of interest. (Of course, we already know about this point.)

The restriction of f to $y = 0$ is

$$f|_{y=0}(x) = x^2.$$

We compute

$$\frac{d}{dx}(f|_{y=0}) = 2x.$$

This derivative is zero when $x = 0$. The corresponding point in our region is $(0,0)$. This is a point of interest. (We already know about this point of interest.)

The restriction of f to $y = 2 - 2x$ is

$$f|_{y=2-2x}(x) = x^2 + (2-2x)^2.$$

We compute

$$\frac{d}{dx}(f|_{y=2-2x}) = 2x + 2(2-2x)(-2) = 10x - 8.$$

This derivative is zero when $x = \frac{4}{5}$. The corresponding point in our region is $\underline{(\frac{4}{5}, \frac{2}{5})}$. This is a point of interest.

Now we calculate f at each point interest. The maximum value of f on the points of interest is the maximum of f on our region. The minimum value of f on the points of interest is the minimum of f on our region.

Point of Interest	f(Point of Interest)
$(0,0)$	$0^2+(0)^2=0$
$(0,2)$	$2(0)^2+(2)^2=4$
$(1,0)$	$(1)^2+(0)^2=1$
$(\frac{4}{5},\frac{2}{5})$	$\frac{16}{25}+\frac{4}{25}=\frac{4}{5}$

The maximum of f on our region occurs at $(0,2,4)$.
The minimum of f on our region occurs at $(0,0,0)$.

Picture for 14.7 Number 37

Our region is