14.7, number 31: **Find the absolute maxima and absolute minima of $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant.**

Answer: There is a picture of the region on the next page. Observe that $\underline{(0,0)}$, $\underline{(0,2)}$, and $\underline{(1,2)}$ are end points of the boundary. These are points of interest. After we have found all of the points of interest, we will evaluate f at each of these points.

Any point in the region where both partial derivatives are zero is a point of interest. We see that $f_x = 4x - 4$ and $f_y = 2y - 4$. So $(1,2)$ is the only point in the region where both partial derivatives are zero. This is a point of interest. (Of course, we already know about this point.)

Now we look at f restricted to each part of the boundary.

The restriction of f to $x = 0$ is

$$f|_{x=0}(y) = y^2 - 4y + 1.$$

We compute

$$\frac{d}{dy}(f|_{x=0}) = 2y - 4.$$

This derivative is zero when $y = 2$. The corresponding point in our region is $(0,2)$. This is a point of interest. (Of course, we already know about this point.)

The restriction of f to $y = 2$ is

$$f|_{y=2}(x) = 2x^2 - 4x + 4 - 8 + 1.$$

We compute

$$\frac{d}{dx}(f|_{y=2}) = 4x - 4.$$

This derivative is zero when $x = 1$. The corresponding point in our region is $(1,2)$. This is a point of interest. (We already know about this point of interest.)

The restriction of f to $y = 2x$ is

$$f|_{y=2x}(x) = 2x^2 - 4x + (2x)^2 - 4(2x) + 1; \quad \text{or} \quad f|_{y=2x}(x) = 6x^2 - 12x + 1.$$

We compute

$$\frac{d}{dx}(f|_{y=x}) = 12x - 12.$$

This derivative is zero when $x = 1$. The corresponding point in our region is $(1,2)$. This is a point of interest. (Of course, we already know about this point.)

Now we calculate f at each point interest. The maximum value of f on the points of interest is the maximum of f on our region. The minimum value of f on the points of interest is the minimum of f on our region.

Point of Interest	f(Point of Interest)
$(0,0)$	$2(0)^2 - 4(0) + (0)^2 - 4(0) + 1 = 1$
$(0,2)$	$2(0)^2 - 4(0) + (2)^2 - 4(2) + 1 = -3$
$(1,2)$	$2(1)^2 - 4(1) + (2)^2 - 4(2) + 1 = -5$

The maximum of f on our region occurs at $(0,0,1)$.
The minimum of f on our region occurs at $(1,2,-5)$.

Picture for 14.7 Number 31

Our Region is