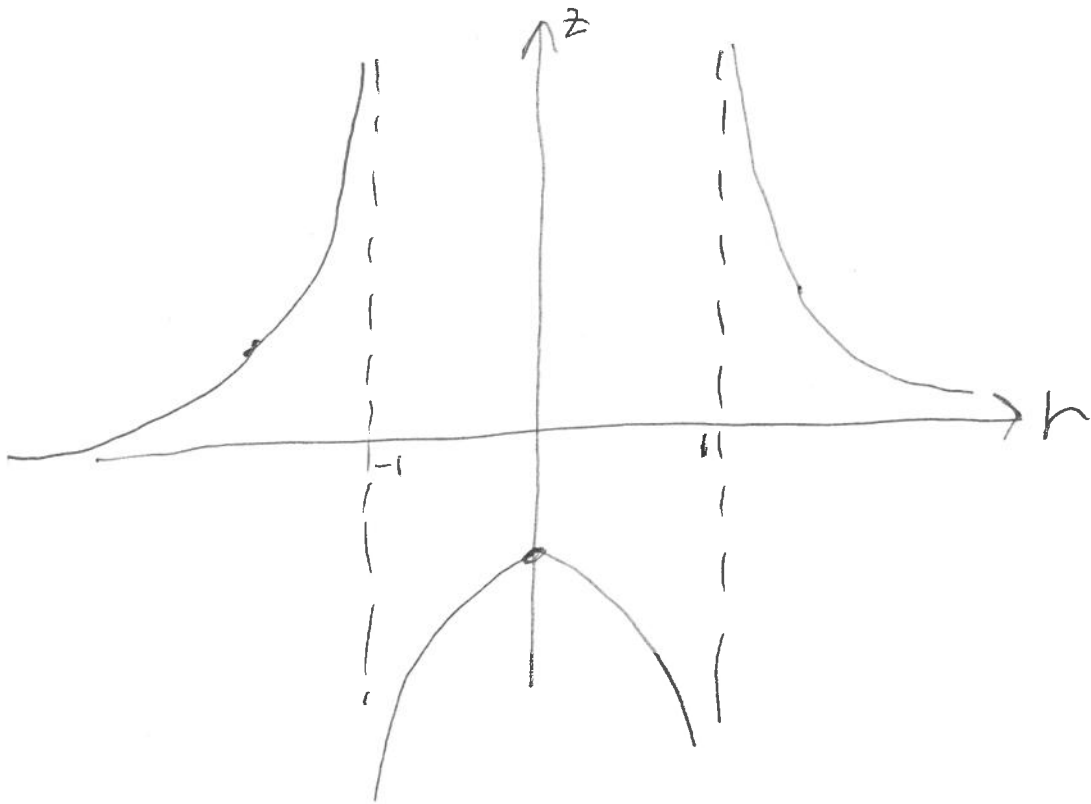14.7, number 21: **Find all local maxima, local minima, and saddle points of** $f(x, y) = \frac{1}{x^2+y^2-1}$.

Answer: We compute $f_x = \frac{-2x}{(x^2+y^2-1)^2}$ and $f_y = \frac{-2y}{(x^2+y^2-1)^2}$. Both partial derivatives are zero at $(0, 0)$ and nowhere else. The second derivative test looks unpleasant, but it is easy to see that

$$\boxed{f(0, 0) = -1 \text{ is a local maximum point on the graph of } z = f(x, y).}$$

Let $r^2 = x^2 + y^2$. It is clear that $(r, z) = (0, -1)$ is a local maximum of $z = \frac{1}{r^2-1}$, whose graph appears on the next page.

Picture for 14.7   Number 21

$$Z = \frac{1}{r^2 - 1}$$