14.7, number 1: **Find all local maxima, local minima, and saddle points of $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.**

Answer: We compute

$$\begin{aligned} f_x &= 2x + y + 3 \\ f_y &= x + 2y - 3 \\ f_{xx} &= 2 \\ f_{xy} &= 1 \\ f_{yy} &= 2 \end{aligned}$$

We see that f_x and f_y are zero when both equations:

$$\begin{cases} x + 2y - 3 = 0 \\ 2x + y + 3 = 0 \end{cases} \tag{1}$$

are satisfied. The solution set is not changed if we replace Equation 2 by Equation 2 minus 2 times Equation 1:

$$\begin{cases} x + 2y - 3 = 0 \\ \quad - 3y + 9 = 0 \end{cases}$$

We see that $y = 3$ and $x = -3$. (By the way, we check that $(-3,3)$ really is the solution of (1).)

We apply the second derivative test at the critical point $(-3,3)$. We see that $(f_{xx}f_{yy} - {f_{xy}}^2)|_{(-3,3)} = 2 \cdot 2 - 1$, which is positive; so we know that $(-3,3)$ is a local minimum or a local maximum (and not a saddle point). We see that $f_{xx}|_{(-3,3)} = 2$, which is positive.

We conclude that $(-3, 3, f(-3,3))$ is a local minimum point on the surface.