

14.6, number 15: Let C be the curve which is the intersection of the two surfaces $x + y^2 + 2z = 4$ and $x = 1$. Give parametric equations for the line that is tangent to C at $(1, 1, 1)$.

Answer: We want the line in the plane $x = 1$ which is tangent to

$$1 + y^2 + 2z = 4$$

when $(y, z) = (1, 1)$. Let's just do this problem as a Math 141 problem. The slope of the tangent line is $\frac{dz}{dy}|_{(y,z)=(1,1)}$. Take the derivative of $y^2 + 2z = 3$ with respect to y to see that $2y + 2\frac{dz}{dy} = 0$. Plug $(1, 1)$ in for (y, z) to see $2 + 2\frac{dz}{dy}|_{(y,z)=(1,1)} = 0$; thus, $\frac{dz}{dy}|_{(y,z)=(1,1)} = -1$. We want the line

$$x = 1, \quad (z - 1) = -(y - 1)$$

or

$$\boxed{x = 1, \quad y + z = 2}$$

You can parameterize this line however you like; for example:

$$\boxed{x = 1, \quad y = t, \quad z = 2 - t.}$$