

14.6, number 1: Find the equation of the plane tangent to $x^2 + y^2 + z^2 = 3$ at the point $P = (1, 1, 1)$. Find parametric equations for the line normal to $x^2 + y^2 + z^2 = 3$ at the point $P = (1, 1, 1)$.

Answer: Gradients are perpendicular to level sets! The given equation is a level set of $f(x, y, z) = x^2 + y^2 + z^2$. We want the line through $(1, 1, 1)$ which is parallel to $\vec{\nabla} f|_{(1,1,1)}$. We compute $\vec{\nabla} f = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$ and $\vec{\nabla} f|_{(1,1,1)} = 2\vec{i} + 2\vec{j} + 2\vec{k}$.

The tangent plane is the plane through $(1, 1, 1)$ perpendicular to $2\vec{i} + 2\vec{j} + 2\vec{k}$:

$$2(x - 1) + 2(y - 1) + 2(z - 1) = 0.$$

The normal line is the line through $(1, 1, 1)$ parallel to $2\vec{i} + 2\vec{j} + 2\vec{k}$:

$$x = 1 + 2t, \quad y = 1 + 2t, \quad z = 1 + 2t.$$