14.5, number 6: **Find the gradient of** $f(x,y) = \arctan\left(\frac{\sqrt{x}}{y}\right)$ **at the point** $P = (4,-2)$**. Draw the level set of** f **that passes through** P**. Draw the gradient; put its tail on** P**.**

Answer: Recall $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$. We compute

$$\begin{aligned}\vec{\nabla} f &= f_x \vec{i} + f_y \vec{j} \\ &= \frac{\frac{1}{2\sqrt{x}y}}{1+(\frac{\sqrt{x}}{y})^2}\vec{i} + \frac{-\frac{\sqrt{x}}{y^2}}{1+(\frac{\sqrt{x}}{y})^2}\vec{j}\end{aligned}$$

Multiply top and bottom by y^2 to obtain

$$= \frac{\frac{y}{2\sqrt{x}}\vec{i} - \sqrt{x}\vec{j}}{y^2+x}.$$

So,

$$(\vec{\nabla} f)|_{(4,-2)} = \frac{\frac{-2}{2\sqrt{4}}\vec{i} - \sqrt{4}\vec{j}}{4+4} = \boxed{\frac{-\vec{i} - 4\vec{j}}{16}}.$$

Observe that $f(4,-2) = \arctan\left(\frac{\sqrt{4}}{-2}\right) = \arctan(-1) = -\frac{\pi}{4}$. So, we are supposed to draw the level set

$$\frac{-\pi}{4} = \arctan\left(\frac{\sqrt{x}}{y}\right).$$

Take the tangent of each side:

$$-1 = \left(\frac{\sqrt{x}}{y}\right).$$

Square both sides $y^2 = x$. The picture is on the next page.