14.5, number 25: **Let $f(x,y) = x^2 + y^2$. Consider the level set $f(x,y) = 4$ and the point $P = (\sqrt{2}, \sqrt{2})$ on that level set. Sketch**

(a) **the level set $f(x,y) = 4$,**

(b) $\overrightarrow{\nabla} f|_P$ **(Put the tail of the gradient on P.), and**

(c) **the line tangent to the level set $f(x,y) = 4$ at P.**

What is the equation of the line tangent to the level set $f(x,y) = 4$ at P?

Answer: There is a picture on the next page. The picture shows (a), (b), and (c). Observe that

$$\overrightarrow{\nabla} f = 2x\,\overrightarrow{\boldsymbol{i}} + 2y\,\overrightarrow{\boldsymbol{j}} \quad \text{and} \quad \overrightarrow{\nabla} f|_P = 2\sqrt{2}\,\overrightarrow{\boldsymbol{i}} + 2\sqrt{2}\,\overrightarrow{\boldsymbol{j}}\,.$$

The point (x,y) is on the tangent line if and only if the vector

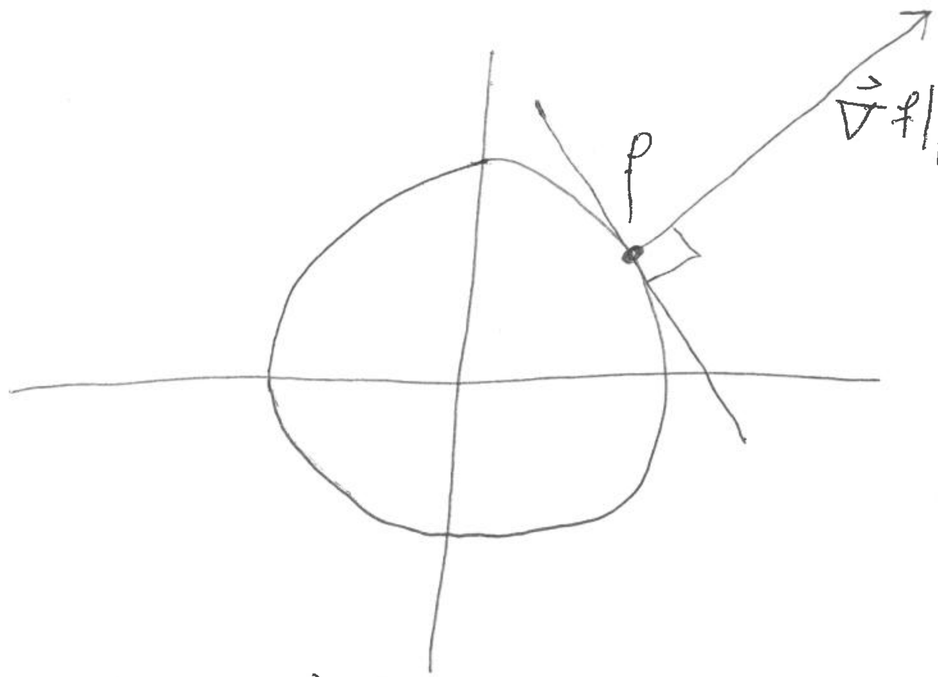
$$\overrightarrow{(\sqrt{2}, \sqrt{2})(x,y)}$$

is perpendicular to $\overrightarrow{\nabla} f|_P$. In other words, (x,y) is on the tangent line if and only if

$$\begin{gathered}
\left((x-\sqrt{2})\,\overrightarrow{\boldsymbol{i}} + (y-\sqrt{2})\,\overrightarrow{\boldsymbol{j}}\right) \cdot (2\sqrt{2}\,\overrightarrow{\boldsymbol{i}} + 2\sqrt{2}\,\overrightarrow{\boldsymbol{j}}\,) = 0 \\
2\sqrt{2}(x-\sqrt{2}) + 2\sqrt{2}(y-\sqrt{2}) = 0. \\
(x-\sqrt{2}) + (y-\sqrt{2}) = 0. \\
\boxed{x + y = 2\sqrt{2}}
\end{gathered}$$

(Of course, you could also use First Semester Calculus techniques to get this answer.)

Picture for 14.5 Number 25



The circle is $x^2+y^2=4$

P is the point $(\sqrt{2}, \sqrt{2})$

$\vec{\nabla} f|_P = 2\sqrt{2}\,\vec{i} + 2\sqrt{2}\,\vec{j}$

The line is tangent to the circle at P.