

14.5, number 19: Let $f(x, y) = x^2 + xy + y^2$ and $P = (-1, 1)$.

- (a) Which direction gives the largest directional derivative of f at P ?
What is the directional derivative of f at P in that direction?
- (b) Which direction gives the smallest directional derivative of f at P ?
What is the directional derivative of f at P in that direction?

Answer: The gradient of f at P points in the direction of the largest directional derivative of f at P . The length of the directional derivative is the directional derivative of f at P in that direction. We compute

$$\vec{\nabla} f|_P = ((2x+y)\vec{i} + (x+2y)\vec{j})|_{(-1,1)} = (-2+1)\vec{i} + (-1+2)\vec{j} = -\vec{i} + \vec{j}.$$

We conclude that

$-\vec{i} + \vec{j}$ points in the direction of the largest directional derivative of f at the point P and the directional derivative of f in that direction is $\sqrt{2}$. Furthermore $\vec{i} - \vec{j}$ points in the direction of the smallest directional derivative of f at P and the directional derivative in that direction is $-\sqrt{2}$.