14.5, number 11: Let $f(x,y)=2xy-3y^2$, P=(5,5), and $\overrightarrow{v}=4\overrightarrow{i}+3\overrightarrow{j}$. Find $D_{\overrightarrow{v}}f|_P$. (That is, find the directional derivative of f in the direction of \overrightarrow{v} at the point P.)

Answer:

$$D_{\overrightarrow{v}}f|_{P} = \overrightarrow{\nabla}f|_{P} \cdot \frac{\overrightarrow{v}}{|\overrightarrow{v}|}$$

$$= (2y\overrightarrow{i} + (2x - 6y)\overrightarrow{j})|_{(5,5)} \cdot \frac{4\overrightarrow{i} + 3\overrightarrow{j}}{\sqrt{16 + 9}}$$

$$= (10\overrightarrow{i} - 20\overrightarrow{j}) \cdot \frac{4\overrightarrow{i} + 3\overrightarrow{j}}{5}$$

$$= (2\overrightarrow{i} - 4\overrightarrow{j}) \cdot (4\overrightarrow{i} + 3\overrightarrow{j})$$

$$= 8 - 12 = \boxed{-4}$$