14.2, number 43: **Show that** $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$ **does not exist.**

Answer: We compute

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y\,=\,0}} \frac{x^4-y^2}{x^4+y^2} = \lim_{x\to 0}\frac{x^4}{x^4} = \lim_{x\to 0} 1 = 1$$

and

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y\,=\,x^2}} \frac{x^4-y^2}{x^4+y^2} \lim_{x\to 0}\frac{x^4-(x^2)^2}{x^4+(x^2)^2} = \lim_{x\to 0}\frac{0}{2x^2} = \lim_{x\to 0} 0 = 0.$$

Two different aproaches to $(0,0)$ gave different values for $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$.

We conclude that $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$ does not exist.