14.2, number 41: **Show that**

$$\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}}$$

does not exist.

Answer:

When x is positive $\sqrt{x^2} = x$; however, when x is negative, $\sqrt{x^2} = -x$. We compute

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0 \text{ and } x \text{ is positive}}} -\frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to 0^+} -\frac{x}{\sqrt{x^2}} = \lim_{x\to 0^+} -\frac{x}{x} = \lim_{x\to 0^+} -1 = -1.$$

On the other hand,

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0 \text{ and } x \text{ is negative}}} -\frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to 0^-} -\frac{x}{\sqrt{x^2}} = \lim_{x\to 0^+} -\frac{x}{-x} = \lim_{x\to 0^+} 1 = 1.$$

Two different aproaches to $(0,0)$ gave different values for

$$\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}}.$$

$$\boxed{\text{We conclude that } \lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}} \text{ does not exist.}}$$