14.1, number 47: Let  $f(x, y) = \sqrt{x^2 + y^2 + 4}$ .

- (a) Graph the surface z = f(x, y).
- (b) Draw a few level sets of *f*.

## Answer:

We have graphed  $z = \sqrt{x^2 + y^2 + 4}$  before! Square both sides and get  $z^2 = x^2 + y^2 + 4$ , which is a hyperboloid of two sheets with the *z*-axis in the middle. (It looks like one drew the hyperbola  $z^2 - y^2 = 4$  in the *yz*-plane and then rotated it about the *z*-axis.

Of course,  $z = \sqrt{x^2 + y^2 + 4}$  is the top sheet of the hyperboloid, because z is always at least 2.

The level sets of f are the origin and a bunch of concentric circles with center at the origin. The origin is the level set f(x, y) = 2; the circle of radius 1 is the level set for  $f(x, y) = \sqrt{5}$ , etc.

There is a picture on the next page.

