13.3, number 9: **Find the point on the curve**

$$\vec{\boldsymbol{r}}(t) = (5\sin t)\,\vec{\boldsymbol{i}} + (5\cos t)\,\vec{\boldsymbol{j}} + 12t\,\vec{\boldsymbol{k}}$$

at a distance 26π **units along the curve from the point** $(0, 5, 0)$ **in the direction corresponding to increasing** t **values.**

Answer: An object moving on the curve stands at the point $(0, 5, 0)$ at time $t = 0$. We will find the distance traveled between time $t = 0$ and time $t = t_0$. (We take t_0 to be positive.) Then we will find the value of t_0 that makes the distance traveled by the object between $t = 0$ and $t = t_0$ equal to 26π. The answer is the position of the object at time equals t_0.

The distance traveled by the object between $t = 0$ and $t = t_0$ is

$$\begin{aligned}
\int_0^{t_0} |\vec{\boldsymbol{r}}'(t)|dt &= \int_0^{t_0} |5\cos t\,\vec{\boldsymbol{i}} - 5\sin t\,\vec{\boldsymbol{j}} + 12\,\vec{\boldsymbol{k}}|dt \\
&= \int_0^{t_0} \sqrt{25\cos^2 t + 25\sin^2 t + 144}dt \\
&= \int_0^{t_0} \sqrt{25 + 144}dt \\
&= 13t_0
\end{aligned}$$

The object has traveled 26π units, when $26\pi = 13t_0$. That is $t_0 = 2\pi$. The position of the object at time $t_0 = 2\pi$ is $(5\sin(2\pi), 5\cos(2\pi), 12(2\pi))$, which is

$$\boxed{(0, 5, 24\pi)}.$$