13.2, number 32: **The picture on the next page shows an experiment with two marbles. Marble A was launched toward marble B with launch angle α and initial speed v_0. At the same instant, marble B was released to fall from rest at $R\tan\alpha$ units directly above a spot R units downrange from A. The marbles were found to collide regardless of the value of v_0. Was this mere coincidence, or must this happen? Give reasons for your answer.**

Answer: Let $\vec{\boldsymbol{r}}_A(t)$ be the position vector of marble A at time t and $\vec{\boldsymbol{r}}_B(t)$ be the position vector of marble B at time t. We will find the time that causes the x-coordinate of marble A to be R. Then we will find the y coordinate of both marbles at that time. If both marbles have the same y-coordinate when marble A has x-coordinate R, then the marbles have collided. Otherwise, they did not collide.

For marble A:

$$\begin{aligned}\vec{\boldsymbol{r}}''_A(t) &= -g\,\vec{\boldsymbol{j}}\\ \vec{\boldsymbol{r}}'_A(0) &= v_0\cos\alpha\,\vec{\boldsymbol{i}} + v_0\sin\alpha\,\vec{\boldsymbol{j}}\\ \vec{\boldsymbol{r}}_A(0) &= 0\end{aligned}$$

Integrate twice and evaluate the constants to learn

$$\vec{\boldsymbol{r}}_A(t) = (v_0\cos\alpha)t\,\vec{\boldsymbol{i}} + \big((v_0\sin\alpha)t - \frac{g}{2}t^2\big)\,\vec{\boldsymbol{j}}$$

For marble B:

$$\begin{aligned}\vec{\boldsymbol{r}}''_B(t) &= -g\,\vec{\boldsymbol{j}}\\ \vec{\boldsymbol{r}}'_B(0) &= 0\\ \vec{\boldsymbol{r}}_B(0) &= R\,\vec{\boldsymbol{i}} + R\tan\alpha\,\vec{\boldsymbol{j}}\end{aligned}$$

Integrate twice and evaluate the constants to learn

$$\vec{\boldsymbol{r}}_B(t) = R\,\vec{\boldsymbol{i}} + \big(R\tan\alpha - \frac{g}{2}t^2\big)\,\vec{\boldsymbol{j}}$$

The x-coordinate of marble A is equal to R, when $(v_0\cos\alpha)t = R$; in other words, when $t = \frac{R}{v_0\cos\alpha}$.

The y-coordinate of marble A when $t = \frac{R}{v_0\cos\alpha}$ is

$$\begin{aligned}&(v_0\sin\alpha)\left(\frac{R}{v_0\cos\alpha}\right) - \frac{g}{2}\left(\frac{R}{v_0\cos\alpha}\right)^2\\ &= R\tan\alpha - \frac{g}{2}\left(\frac{R}{v_0\cos\alpha}\right)^2\end{aligned}$$

The y-coordinate of marble B when $t = R/(v_0\cos\alpha)$ is

$$R\tan\alpha - \frac{g}{2}\left(\frac{R}{v_0\cos\alpha}\right)^2.$$

THE MARBLES COLLIDED.

Picture for Section 13.2 Number 32