13.2, number 23: **A projectile is fired at a speed of 840 m/sec at an angle of 60 degrees. How long will it take to get 21 km down range?**

Answer:

Let $\vec{\boldsymbol{r}}(t) = x(t)\,\vec{\boldsymbol{i}} + y(t)\,\vec{\boldsymbol{j}}$ be the position vector of the projectile at time t, where time is measured in seconds and distance is measured in meters. The acceleration of the object is $\vec{\boldsymbol{r}}''(t) = -g\,\vec{\boldsymbol{j}} = -9.8\,\vec{\boldsymbol{j}}$. We know that $\vec{\boldsymbol{r}}'(0) = 840(\frac{1}{2}\,\vec{\boldsymbol{i}} + \frac{\sqrt{3}}{2}\,\vec{\boldsymbol{j}})$. (If necessary look at the picture on the next page.)

We integrate and evaluate the constants to see that

$$\vec{\boldsymbol{r}}'(t) = 420\,\vec{\boldsymbol{i}} + (420\sqrt{3} - 9.8t)\,\vec{\boldsymbol{j}}\,.$$

We integrate again and evaluate the constants to see that

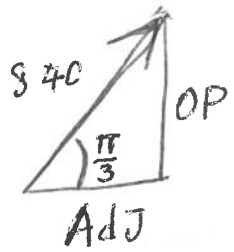
$$\vec{\boldsymbol{r}}(t) = 420t\,\vec{\boldsymbol{i}} + (420\sqrt{3}t - 4.9t^2)\,\vec{\boldsymbol{j}}\,.$$

Observe that $x(t) = 21,000$ when

$$420t = 21,000$$

$$t = \frac{21,000}{420} = \frac{(21)(100)}{42} = \frac{100}{2} = \boxed{50 \text{ seconds}}.$$

Picture for 13.2 Number 23



$\frac{1}{2} = \cos\frac{\pi}{3} = \frac{AdJ}{840}$ $\quad \therefore AdJ = \frac{1}{2}(840)$

$\frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} = \frac{OP}{840}$ $\quad \therefore OP = \frac{\sqrt{3}}{2}(840)$

$\therefore r'(0) = 840\left(\frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}\right)$