13.2, number 21: **At time $t = 0$, a particle is located at the point $(1, 2, 3)$. It travels in a straight line to the point $(4, 1, 4)$, has speed 2 at $(1, 2, 3)$ and has constant acceleration $3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$. Find an equation for the position vector $\vec{\boldsymbol{r}}(t)$ of the particle at time t.**

Answer: We are told $\vec{\boldsymbol{r}}(0) = \vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}}$, $\vec{\boldsymbol{r}}'(0)$ has length 2 and points in the direction of $\overrightarrow{(1, 2, 3)(4, 1, 4)}$, and $\vec{\boldsymbol{r}}''(t) = 3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$.

Observe that $\vec{\boldsymbol{r}}'(0) = \frac{2}{\sqrt{3^2+1+1}}(3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}) = \frac{2}{\sqrt{11}}(3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}})$.

We integrate $\vec{\boldsymbol{r}}''(t)$ to find $\vec{\boldsymbol{r}}'(t)$ and then use the value of $\vec{\boldsymbol{r}}'(0)$ to evaluate the constant. Then we integrate $\vec{\boldsymbol{r}}'(t)$ to find $\vec{\boldsymbol{r}}(t)$ and then use the value of $\vec{\boldsymbol{r}}(0)$ to evaluate the constant.

$$\begin{aligned}\vec{\boldsymbol{r}}'(t) &= \int \vec{\boldsymbol{r}}''(t)dt \\ &= \int 3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}\,dt \\ &= 3t\vec{\boldsymbol{i}} - t\vec{\boldsymbol{j}} + t\vec{\boldsymbol{k}} + \vec{c_1}\end{aligned}$$

$$\frac{2}{\sqrt{11}}(3\vec{\boldsymbol{i}} - 1\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}) = \vec{\boldsymbol{r}}'(0) = 3(0)\vec{\boldsymbol{i}} - (0)\vec{\boldsymbol{j}} + (0)\vec{\boldsymbol{k}} + \vec{c_1}$$

Thus

$$\vec{\boldsymbol{r}}'(t) = 3t\vec{\boldsymbol{i}} - t\vec{\boldsymbol{j}} + t\vec{\boldsymbol{k}} + \frac{2}{\sqrt{11}}(3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}})$$

$$\vec{\boldsymbol{r}}'(t) = (t + \frac{2}{\sqrt{11}})(3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}})$$

$$\begin{aligned}\vec{\boldsymbol{r}}(t) &= \int \vec{\boldsymbol{r}}'(t)dt \\ &= (\frac{t^2}{2} + \frac{2t}{\sqrt{11}})(3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}) + \vec{c_2}\end{aligned}$$

$$\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}} = \vec{\boldsymbol{r}}(0) = \vec{c_2}$$

Thus,

$$\boxed{\vec{\boldsymbol{r}}(t) = (\frac{t^2}{2} + \frac{2t}{\sqrt{11}})(3\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}) + \vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}}}$$