13.2, number 17: **Find** $\vec{\boldsymbol{r}}(t)$ **if** $\frac{d^2\vec{\boldsymbol{r}}}{dt^2} = -32\,\vec{\boldsymbol{k}}$**,** $\vec{\boldsymbol{r}}(0) = 100\,\vec{\boldsymbol{k}}$**, and** $\left.\frac{d\vec{\boldsymbol{r}}}{dt}\right|_{t=0} = 8\,\vec{\boldsymbol{i}} + 8\,\vec{\boldsymbol{j}}$**.**

Answer: Integrate $\frac{d^2\vec{\boldsymbol{r}}}{dt^2} = -32\,\vec{\boldsymbol{k}}$ to learn that

$$\frac{d\vec{\boldsymbol{r}}}{dt} = -32t\,\vec{\boldsymbol{k}} + \vec{\boldsymbol{c}}.$$

Plug in $t = 0$ to learn that $8\,\vec{\boldsymbol{i}} + 8\,\vec{\boldsymbol{j}} = \left.\frac{d\vec{\boldsymbol{r}}}{dt}\right|_{t=0} = -32(0) + \vec{\boldsymbol{c}}$**.** Thus,

$$8\,\vec{\boldsymbol{i}} + 8\,\vec{\boldsymbol{j}} = \vec{\boldsymbol{c}}$$

and

$$\frac{d\vec{\boldsymbol{r}}}{dt} = -32t\,\vec{\boldsymbol{k}} + \vec{\boldsymbol{c}} = 8\,\vec{\boldsymbol{i}} + 8\,\vec{\boldsymbol{j}} - 32t\,\vec{\boldsymbol{k}}.$$

Integrate again,

$$\vec{\boldsymbol{r}} = 8t\,\vec{\boldsymbol{i}} + 8t\,\vec{\boldsymbol{j}} - 16t^2\,\vec{\boldsymbol{k}} + \vec{c_1}.$$

Plug in $\vec{\boldsymbol{r}}(0) = 100\,\vec{\boldsymbol{k}}$ to learn

$$100\,\vec{\boldsymbol{k}} = \vec{\boldsymbol{r}}(0) = \vec{c_1}.$$

Thus,

$$\boxed{\vec{\boldsymbol{r}}(t) = 8t\,\vec{\boldsymbol{i}} + 8t\,\vec{\boldsymbol{j}} + (100 - 16t^2)\,\vec{\boldsymbol{k}}.}$$