13.1, number 9: The position vector of an object at time t is $\overrightarrow{r}(t) = (\sin t) \overrightarrow{i} + (\cos t) \overrightarrow{j}$. Find the velocity vector and the acceleration vector of the object at time $\pi/4$ and time $\pi/2$. Draw these vectors on the graph of the path of the object; put the tail of each vector on the position of the object at the given times. (Of course, the object is traveling on the circle $x^2 + y^2 = 1$.)

Answer:

$$\vec{\boldsymbol{v}}(t) = (\cos t)\vec{\boldsymbol{i}} - (\sin t)\vec{\boldsymbol{j}}, \quad \vec{\boldsymbol{a}}(t) = -(\sin t)\vec{\boldsymbol{i}} - (\cos t)\vec{\boldsymbol{j}},$$
$$\vec{\boldsymbol{v}}(\frac{\pi}{4}) = (\cos \frac{\pi}{4})\vec{\boldsymbol{i}} - (\sin \frac{\pi}{4})\vec{\boldsymbol{j}} = \boxed{\frac{\sqrt{2}}{2}\vec{\boldsymbol{i}} - \frac{\sqrt{2}}{2}\vec{\boldsymbol{j}}},$$
$$\vec{\boldsymbol{a}}(\frac{\pi}{4}) = -(\sin \frac{\pi}{4})\vec{\boldsymbol{i}} - (\cos \frac{\pi}{4})\vec{\boldsymbol{j}} = \boxed{-\frac{\sqrt{2}}{2}\vec{\boldsymbol{i}} - \frac{\sqrt{2}}{2}\vec{\boldsymbol{j}}},$$
$$\vec{\boldsymbol{v}}(\frac{\pi}{2}) = (\cos \frac{\pi}{2})\vec{\boldsymbol{i}} - (\sin \frac{\pi}{2})\vec{\boldsymbol{j}} = \boxed{0\vec{\boldsymbol{i}} - 1\vec{\boldsymbol{j}}},$$
$$\vec{\boldsymbol{a}}(\frac{\pi}{2}) = -(\sin \frac{\pi}{2})\vec{\boldsymbol{i}} - (\cos \frac{\pi}{2})\vec{\boldsymbol{j}} - 1\vec{\boldsymbol{i}} - 0\vec{\boldsymbol{j}}.$$

Pictures for 13,1 #9

