

13.1, number 7: The position vector of an object at time t is $\vec{r}(t) = e^t \vec{i} + \frac{2}{9}e^{2t} \vec{j}$. Eliminate t and find an equation involving only x and y which gives the path of the object. Find the object's velocity and acceleration vectors at time $t = \ln 3$.

Answer: We are given $x = e^t$, $y = \frac{2}{9}e^{2t}$. We drop the first equation into the second equation: $y = \frac{2}{9}x^2$. We compute

$$\vec{v}(t) = \vec{r}'(t) = e^t \vec{i} + \frac{4}{9}e^{2t}$$

and

$$\vec{a}(t) = \vec{v}'(t) = e^t \vec{i} + \frac{8}{9}e^{2t}.$$

We conclude that

$$\vec{v}(\ln 3) = e^{\ln 3} \vec{i} + \frac{4}{9}(e^{\ln 3})^2 \vec{j} = \boxed{3 \vec{i} + 4 \vec{j}}$$

and

$$\vec{a}(\ln 3) = e^{\ln 3} \vec{i} + \frac{8}{9}(e^{\ln 3})^2 \vec{j} = \boxed{3 \vec{i} + 8 \vec{j}}.$$