

13.1, number 39: A particle moves along the top of the parabola $y^2 = 2x$ from left to right at a constant speed of 5 units per second. Find the velocity vector of the particle as it moves through the point $(2, 2)$.

Answer: The velocity vector has the same direction as the line tangent to the curve at the point $(2, 2)$. We use implicit differentiation to calculate $2y \frac{dy}{dx} = 2$. Plug in the point $(2, 2)$:

$$2(2) \frac{dy}{dx} \Big|_{(x,y)=(2,2)} = 2.$$

So,

$$\frac{dy}{dx} \Big|_{(x,y)=(2,2)} = \frac{1}{2}.$$

The velocity vector points roughly rightward and has slope $\frac{1}{2}$; so the velocity vector has the same direction as $2\vec{i} + 1\vec{j}$ and the length of the velocity vector is 5. The velocity vector is

$$\boxed{\frac{5}{\sqrt{1^2 + 2^2}}(2\vec{i} + 1\vec{j})}.$$