13.1, number 37e: **The position vector of a particle at time t is given by $\overrightarrow{\boldsymbol{r}}(t) = \cos(t^2)\,\overrightarrow{\boldsymbol{i}} + \sin(t^2)\,\overrightarrow{\boldsymbol{j}}$, for $0 \le t$. (Of course the particle moves on the circle $x^2 + y^2 = 1$.)**

i) **Does the particle have a constant speed? If so, what is it?**

ii) **Is the particle's acceleration always orthogonal to its velocity vector?**

iii) **Does the particle move clock-wise or counterclockwise around the circle?**

iv) **Is the particle initially located at the point $(1, 0)$?**

Answer:

We calculate

$$\overrightarrow{\boldsymbol{v}}(t) = -2t\sin(t^2)\,\overrightarrow{\boldsymbol{i}} + 2t\cos(t^2)\,\overrightarrow{\boldsymbol{j}}$$

$$\overrightarrow{\boldsymbol{a}}(t) = \big(-4t^2\cos(t^2) - 2\sin(t^2)\big)\,\overrightarrow{\boldsymbol{i}} + \big(-4t^2\sin(t^2) + 2\cos(t^2)\big)\,\overrightarrow{\boldsymbol{j}}$$

The speed of the object is

$$|\overrightarrow{\boldsymbol{v}}(t)| = |-2t\sin(t^2)\,\overrightarrow{\boldsymbol{i}} + 2t\cos(t^2)\,\overrightarrow{\boldsymbol{j}}| = \sqrt{4t^2\sin^2(t^2) + 4t^2\cos(\,t^2)}$$
$$= 2t\sqrt{\sin^2(t^2) + \cos^2(t^2)} = 2t.$$

(a) The object does NOT have constant speed.

The dot product of acceleration and velocity is

$$\Big(\big(-4t^2\cos(t^2) - 2\sin(t^2)\big)\,\overrightarrow{\boldsymbol{i}} + \big(-4t^2\sin(t^2) + 2\cos(t^2)\big)\,\overrightarrow{\boldsymbol{j}}\Big) \cdot (-2t\sin(t^2)\,\overrightarrow{\boldsymbol{i}} + 2t\cos(t^2)\,\overrightarrow{\boldsymbol{j}})$$

$$= \big(-4t^2\cos(t^2) - 2\sin(t^2)\big)(-2t\sin(t^2)) + \big(-4t^2\sin(t^2) + 2\cos(t^2)\big)2t\cos(t^2)$$
$$= 8t^3\cos(t^2)\sin(t^2) + 4t\sin^2(t^2) - 8t^3\sin(t^2)\cos(t^2) + 4t\cos^2(t^2)$$
$$= 4t\sin^2(t^2) + 4t\cos^2(t^2) = 4t(\sin^2 t + \cos^2 t) = 4t.$$

Observe that $4t$ is not zero, Conclude that

(b) acceleration and velocity are NOT perpendicular.

(d) The position of the object at $t = 0$ is $(1, 0)$.

The velocity at time 0 of the object is $\overrightarrow{\boldsymbol{v}}(0) = 2\,\overrightarrow{\boldsymbol{j}}$. The object starts at $(1, 0)$ and is moving straight up.

(c) The object is moving ccw.