13.1, number 37b: **The position vector of a particle at time t is given by $\vec{\boldsymbol{r}}(t) = \cos(2t)\,\vec{\boldsymbol{i}} + \sin(2t)\,\vec{\boldsymbol{j}}$, for $0 \le t$. (Of course, the particle moves on the circle $x^2 + y^2 = 1$.)**

i) Does the particle have a constant speed? If so, what is it?

ii) Is the particle's acceleration always orthogonal to its velocity vector?

iii) Does the particle move clock-wise or counterclockwise around the circle?

iv) Is the particle initially located at the point $(1, 0)$?

Answer: We calculate

$$\vec{\boldsymbol{v}}(t) = -2\sin(2t)\,\vec{\boldsymbol{i}} + 2\cos(2t)\,\vec{\boldsymbol{j}}$$

$$\vec{\boldsymbol{a}}(t) = -4\cos(2t)\,\vec{\boldsymbol{i}} - 4\sin(2t)\,\vec{\boldsymbol{j}}$$

The speed of the object is

$$\begin{aligned} |\vec{\boldsymbol{v}}(t)| &= |-2\sin(2t) + 2\cos(2t)| = \sqrt{4\sin^2(2t) + 4\cos^(2t)} \\ &= 2\sqrt{\sin^2(2t) + \cos^2(2t)} = 2. \end{aligned}$$

(a) The object has constant speed 2

The dot product of acceleration and velocity is

$$\begin{aligned} &(-4\cos(2t)\,\vec{\boldsymbol{i}} - 4\sin(2t)\,\vec{\boldsymbol{j}}) \cdot (-2\sin(2t)\,\vec{\boldsymbol{i}} + 2\cos(2t)\,\vec{\boldsymbol{j}}) \\ &= 8\cos(2t)\sin(2t) - 8\sin(2t)\cos(2t) = 0. \end{aligned}$$

(b) Yes, acceleration and velocity are perpendicular.

(d) The position of the object at $t = 0$ is $(1, 0)$.

The velocity at time 0 of the object is $\vec{\boldsymbol{v}}(0) = 2\,\vec{\boldsymbol{j}}$. The object starts at $(1, 0)$ and is moving start up.

(c) The object is moving ccw.