

13.1, number 23: Find parametric equations for the line that is tangent to the curve parameterized by $\vec{r}(t) = (\sin t)\vec{i} + (t^2 - \cos t)\vec{j} + e^t\vec{k}$ at $t = 0$.

Answer: The point on the curve at time $t = 0$ is

$$(\sin(0), 0^2 - \cos(0), e^0) = (0, -1, 1).$$

A vector tangent to the curve (and parallel to the tangent line) at time 0 is $\vec{r}'(0)$. We see that $\vec{r}'(t) = \cos t\vec{i} + (2t + \sin t)\vec{j} + e^t\vec{k}$. Thus $\vec{r}'(0) = \vec{i} + \vec{k}$ and the tangent line is

$$\boxed{x = t, y = -1, z = 1 + t}.$$