

13.1, number 13: The position vector of an object at time t is

$$\vec{r}(t) = (t+1)\vec{i} + (t^2-1)\vec{j} + 2t\vec{k}.$$

Find the object's velocity vector and acceleration vector. Find the object's speed and direction of motion at time $t = 1$. Write $\vec{v}(1)$ as the object's speed at time $t = 1$ times a unit vector.

Answer: We compute $\vec{v}(t) = \vec{r}'(t)$; so

$$\vec{v}(t) = \vec{i} + 2t\vec{j} + 2\vec{k}.$$

Also, $\vec{a}(t) = \vec{v}'(t)$; so,

$$\vec{a}(t) = 2\vec{j}.$$

At $t = 1$, $\vec{v}(1) = \vec{i} + 2\vec{j} + 2\vec{k}$. The speed of the vector at $t = 1$ is $|\vec{v}(1)| = \sqrt{1^2 + 2^2 + 2^2}$.

The speed of the vector at $t = 1$ is 3.

A unit vector that points in the direction of motion at $t = 1$ is $\frac{1}{3}(\vec{i} + 2\vec{j} + 2\vec{k})$.

So

$\vec{v}(1) = 3(\frac{1}{3}(\vec{i} + 2\vec{j} + 2\vec{k}))$ where 3 is the speed and $\frac{1}{3}(\vec{i} + 2\vec{j} + 2\vec{k})$ is a unit vector in the direction of motion.