This problem comes from Spring 2023, Exam 1, number 2.

Find an equation for the plane through the points $P_1 = (1, -1, 2)$, $P_2 = (2, 4, -1)$, and $P_3 = (3, 2, 1)$. Check your answer. Make sure it is correct.

Answer: Observe that

$$\overrightarrow{P_1P_2} = \vec{\boldsymbol{i}} + 5\vec{\boldsymbol{j}} - 3\vec{\boldsymbol{k}} \text{ and } \overrightarrow{P_1P_3} = 2\vec{\boldsymbol{i}} + 3\vec{\boldsymbol{j}} - 1\vec{\boldsymbol{k}}.$$

It follows that

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ 1 & 5 & -3 \\ 2 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 5 & -3 \\ 3 & -1 \end{vmatrix} \vec{\boldsymbol{i}} - \begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} \vec{\boldsymbol{j}} + \begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} \vec{\boldsymbol{k}}$$

$$= 4\vec{\boldsymbol{i}} - 5\vec{\boldsymbol{j}} - 7\vec{\boldsymbol{k}}.$$

The plane through $(1, -1, 2)$ perpendicular to $-4\vec{\boldsymbol{i}} - 5\vec{\boldsymbol{j}} - 7\vec{\boldsymbol{k}}$ is

$$4(x-1) - 5(y+1) - 7(z-2) = 0$$

or

$$\boxed{4x - 5y - 7z = -5}.$$

Check. Plug $(1, -1, 2)$ into the proposed answer:

$$4(1) - 5(-1) - 7(2) = -5\checkmark$$

Plug $(2, 4, -1)$ into the proposed answer:

$$4(2) - 5(4) - 7(-1) = -5\checkmark$$

Plug $(3, 2, 1)$ into the proposed answer:

$$4(3) - 5(2) - 7(1) = -5\checkmark$$