This problem comes from Fall 2022, Exam 1, number 1.

Find an equation for the plane through the points $P_1 = (1,2,3)$, $P_2 = (1,1,1)$, and $P_3 = (-1,0,1)$. Check your answer. Make sure it is correct.

Answer: The vector $\overrightarrow{P_1P_2}$ is equal to $-\overrightarrow{\boldsymbol{j}} - 2\overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{P_1P_3} = -2\overrightarrow{\boldsymbol{i}} - 2\overrightarrow{\boldsymbol{j}} - 2\overrightarrow{\boldsymbol{k}}$. We compute

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\ 0 & -1 & -2 \\ -2 & -2 & -2 \end{vmatrix} = \begin{vmatrix} -1 & -2 \\ -2 & -2 \end{vmatrix} \overrightarrow{\boldsymbol{i}} - \begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix} \overrightarrow{\boldsymbol{j}} + \begin{vmatrix} 0 & -1 \\ -2 & -2 \end{vmatrix} \overrightarrow{\boldsymbol{k}}$$

$$= -2\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} - 2\overrightarrow{\boldsymbol{j}}.$$

The plane through $(1,2,3)$ perpendicular to $-2\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} - 2\overrightarrow{\boldsymbol{j}}$ is

$$-2(x-1) + 4(y-2) - 2(z-3) = 0$$

or

$$-(x-1) + 2(y-2) - (z-3) = 0$$

or

$$\boxed{-x + 2y - z = 0}.$$

Check. The point $(1,2,3)$ satisfies the proposed answer because

$$-1 + 2(2) - 3 = 0.$$

The point $(1,1,1)$ satisfies the proposed answer because

$$-1 + 2 - 1 = 0.$$

The point $(-1,0,1)$ satisfies the proposed answer because $-(-1) - 1 = 0$.