

12.5, number 57: **Find the intersection of the line**

$$\begin{cases} x = 1 - t \\ y = 3t \\ z = 1 + t \end{cases}$$

and the plane $2x - y + 3z = 6$.

Answer: Consider an object moving along the line so that its position at time t is $(1 - t, 3t, 1 + t)$. We first find the time that puts the object on the plane. In other words, find t so that

$$2(1 - t) - (3t) + 3(1 + t) = 6$$

$$2 - 2t - 3t + 3 + 3t = 6$$

$$-2t = 1$$

$$t = -1/2$$

Now we figure out where the object is at time $t = -1/2$:

$$\boxed{\left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}\right)}$$

Check

Observe that our proposed answer is on the plane:

$$2\left(\frac{3}{2}\right) - \left(-\frac{3}{2}\right) + 3\left(\frac{1}{2}\right) = 6.\checkmark$$

Of course our proposed answer is also on the line. (It is where the line is at time $t = -1/2$.)