12.5, number 53: **Find the angle between the planes** $2x + 2y + 2z = 3$ **and** $2x - 2y - z = 5$**. (There are two angles that you might measure. You should measure the smaller of the two angles.)**

Answer: The angel between the planes is exactly the same as the angle between the vectors perpendicular to the planes. The vector $\overrightarrow{\boldsymbol{N}}_1 = 2\overrightarrow{\boldsymbol{i}} + 2\overrightarrow{\boldsymbol{j}} + 2\overrightarrow{\boldsymbol{k}}$ is perpendicular to the first plane. The vector $\overrightarrow{\boldsymbol{N}}_2 = 2\overrightarrow{\boldsymbol{i}} - 2\overrightarrow{\boldsymbol{j}} - \overrightarrow{\boldsymbol{k}}$ to the second plane. Of course, $-\overrightarrow{\boldsymbol{N}}_1$ is also perpendicular to the first plane. Let θ be the angle between $\overrightarrow{\boldsymbol{N}}_1$ and $\overrightarrow{\boldsymbol{N}}_2$. Let θ' be the angle between $-\overrightarrow{\boldsymbol{N}}_1$ and $\overrightarrow{\boldsymbol{N}}_2$. One of these angles will be in the first quadrant and the cosine of this angle will be positive. The other angle will be in the second quadrant and the cosine of this angle will be negative. The answer is the angle with the positive cosine. Use dot product to compute

$$-2 = 4 - 4 - 2 = \overrightarrow{\boldsymbol{N}}_1 \cdot \overrightarrow{\boldsymbol{N}}_2 = |\overrightarrow{\boldsymbol{N}}_1||\overrightarrow{\boldsymbol{N}}_2| \cos\theta = \sqrt{12}\sqrt{9}\cos\theta.$$

Thus

$$\cos\theta = \frac{-2}{6\sqrt{3}} = \frac{-1}{3\sqrt{3}}.$$

It follows that $\cos(\theta') = \frac{1}{3\sqrt{3}}$ and

$$\theta' = \boxed{\arccos\left(\frac{1}{3\sqrt{3}}\right)}$$

is the answer.