

12.5, number 45: Find the distance between the planes $x + 2y + 6z = 1$ and $x + 2y + 6z = 10$.

Answer: Pick a point P_1 on the first plane. Find the point P_2 on the second plane which is closest to P_1 . The answer is the distance from P_1 to P_2 .

We see that $P_1 = (-1, 1, 0)$ is on the first plane. The line through P_1 perpendicular to the first plane is

$$\begin{cases} x = t - 1 \\ y = 2t + 1 \\ z = 6t + 0 \end{cases}$$

The line we made up hits the second plane when

$$(t - 1) + 2(2t + 1) + 6(6t) = 10$$

or $t = \frac{9}{41}$. The line we made up hits the second plane at the point

$$P_2 = \left(\frac{9}{41} - 1, \frac{18}{41} + 1, 6\left(\frac{9}{41}\right)\right)$$

The distance between the planes is the distance from P_1 to P_2 , which is

$$\sqrt{\left(\frac{9}{41}\right)^2 + \left(\frac{18}{41}\right)^2 + \left(\frac{54}{41}\right)^2} = \frac{9\sqrt{1+4+36}}{41} = \boxed{\frac{9\sqrt{41}}{41}}.$$