12.5, number 39: **Find the point on the plane $x + 2y + 2z = 13$ which is closest to the point $(2, -3, 4)$.**

Answer: We find the equations of the line through $(2, -3, 4)$ which is perpendicular to the given plane. Our answer is the point of intersection of the line we make up and the given plane. (There is a picture on the next page.)

The line through $(2, -3, 4)$ parallel to $\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 2\vec{\boldsymbol{k}}$ is

$$\begin{cases} x - 2 = t \\ y + 3 = 2t \\ z - 4 = 2t \end{cases}$$

The line we made up intersects the plane we were given when

$$(t + 2) + 2(2t - 3) + 2(2t + 4) = 13$$

$$9t = 13 - 2 + 6 - 8$$

$$9t = 9$$

$$t = 1$$

When $t = 1$, our line hits the point $(1 + 2, 2 - 3, 2 + 4)$. The point on the plane closest to $(2, -3, 4)$ is $\boxed{(3, -1, 6)}$.

Check.

The proposed answer is on the plane because $3 - 2 + 12 = 13$✓. The vector from $(2, -3, 4)$ to the proposed answer is $\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 2\vec{\boldsymbol{k}}$, which is perpendicular to the given plane. ✓

Picture for 12.5 Number 39